

# Relations between the scales of length, time and mass

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## Abstract

It is considered the model of the homogeneous and isotropic universe. The scale of length is defined via the laboratory scale of time by the motion of photon. This leads to the appearance of the inertial forces. The properties of the space and time are defined both by these inertial forces and by the matter. Within the framework of classical physics, the scales of length, time and mass are related by the special relativity constant  $c$  and by the Newton gravity constant  $G$ . Within the framework of quantum mechanics, the scales of length, time and energy are related by the special relativity constant  $c$  and by the quantum mechanics constant  $\hbar$ . The model meets constraints from the current age of the universe, from the high-redshift supernovae data, and from primordial nucleosynthesis. The model predicts the fractal galaxy distribution with a power index of 2.

## 1 Introduction

According to the general relativity the properties of the space and time are defined by the matter. However in order to specify the scales of length and time it is necessary to introduce inertial reference frames, the rest frame and the frame moving with the velocity  $v$  relative to the rest frame. While defining the scale of time  $t$  in the rest frame, one can introduce the scale of length by means of the moving frame  $R = vt$ . The transition from the rest frame to the moving frame specifies an inertial acceleration  $F_{in} = v/t = v^2/R$ . Thus the introduction of the space and time by means of the inertial frames is accompanied by the introduction of the non-inertial frames. This means that a priori granted cosmological inertial forces must exist in the universe. Hence the properties of the space and time are defined both by the matter and by the inertial forces.

## 2 Theory

### 2.1 The space and time of the universe

Consider the homogeneous and isotropic universe in the laboratory system of reference. Introduce the laboratory time  $t$ . At the fixed moment of the laboratory time  $t = const$ , the motion of photon relative to the rest frame specifies the scale of length

$$a = ct. \tag{1}$$

Along this scale of length, the motion of photon specifies the field of velocities

$$v = \frac{R}{a}c \quad (2)$$

and the corresponding field of accelerations

$$F_{in} = \frac{v}{t} = \frac{v^2}{R} = \frac{c^2}{a^2}R. \quad (3)$$

The acceleration  $F_{in}$  is due to the inertial force arising under the transition from the rest frame to the moving frame. Thus determination of the scale of length by means of photon moving relative to the rest frame leads to the appearance of the inertial acceleration.

Take the volume of the radius  $a$ . The matter homogeneously distributed in this volume specifies the field of accelerations due to gravity

$$F_{gr} = -G\rho R. \quad (4)$$

The acceleration due to gravity  $F_{gr}$  and the inertial acceleration  $F_{in}$  act in the opposite directions. If the density of the matter is equal to

$$\rho = \frac{c^2}{Ga^2} \quad (5)$$

the acceleration due to gravity  $F_{gr}$  balances the inertial acceleration  $F_{in}$ , and the resulting acceleration is equal to zero

$$F = F_{in} + F_{gr} = 0. \quad (6)$$

Thus, at  $t = \text{const}$ , the scale of length  $a$  is the element of the euclidean space, with the gravitational forces due to the matter of the universe balancing the inertial forces due to the motion of photon.

It should be noted that the acceleration  $F = \Lambda c^2 R$  proposed by Einstein [1] coincides with the inertial acceleration given by (3) if to adopt  $\Lambda = a^{-2}$ . Thus we arrive at the model of the universe in the spirit of Einstein. Unlike the conventional point of view when  $\Lambda$ -term is associated with the vacuum, here  $\Lambda$ -term is associated with the cosmological inertial forces. Unlike the Einstein model describing the static universe, here the universe evolves following the linear law (1).

## 2.2 Evolution of the universe within the framework of classical physics

Take the universe as a particle with the radius  $a$  and the mass  $m$ . The values  $a$  and  $m$  are the scales of length and mass at the fixed moment of the laboratory time  $t = \text{const}$ . Let the scale of length  $a$  be the element of the euclidean space. That is the acceleration due to gravity (4) balances the inertial acceleration (3). Then the density of the matter in the universe is given by eq. (5) that corresponds to the closed world with zero total mass. That is the mass of the matter of the universe is equal to the gravity of the universe

$$mc^2 = \frac{Gm^2}{a}. \quad (7)$$

Evolution of the universe is defined by the motion of photon. The law of evolution for the scale of length is given by eq. (1). Since the scale of mass is bounded with the scale of length by the relation (7), from the law of evolution for the scale of length (1) it follows the law of evolution for the scale of mass

$$m = \frac{c^2}{G}a = \frac{c^3}{G}t. \quad (8)$$

Thus, within the framework of classical physics, the scales of length and mass are the linear functions of time which are defined by the constant of special relativity  $c$  and by the constant of the Newton gravity  $G$ . The law of evolution for the scale of length (1) and the law of evolution for the scale of mass (8) give the law of evolution for the density of the matter

$$\rho = \frac{3m}{4\pi a^3} = \frac{3c^2}{4\pi G a^2} = \frac{3}{4\pi G t^2}. \quad (9)$$

## 2.3 Evolution of the universe within the framework of quantum mechanics

Consider the evolution of the universe within the framework of quantum mechanics. At  $t = \text{const}$ , compare the universe of the mass  $m$  and the radius  $a$  with a wave of the frequency

$$\omega \equiv \frac{E}{\hbar} = \frac{mc^2}{\hbar} \quad (10)$$

and of the wave number

$$k \equiv \frac{\omega}{c} = \frac{1}{a}. \quad (11)$$

The frequency and the wave number are the scales of energy and length respectively.

Due to the Heisenberg uncertainty principle the scale of energy (frequency) and the lifetime of the universe are related as

$$Et = \hbar, \quad (12)$$

the scale of momentum projection  $p_x$  and the scale of length  $a_x$  are related as

$$p_x a_x = \hbar. \quad (13)$$

Thus, within the framework of quantum mechanics, the evolution of the universe is governed by the Heisenberg uncertainty principle. The evolution laws for the scales of energy (frequency) and length are defined by the constant of special relativity  $c$  and by the constant of quantum mechanics  $\hbar$ . The evolution law for scale of length is linear like in the classical physics. Unlike the linear law of evolution for the scale of mass in the classical physics, in the quantum mechanics the law of evolution for the scale of energy (frequency) is inversely linear. Hence one can put the scale of energy (frequency) into correspondence to the scale of mass only in a unique moment of time. Such a moment when the scale of energy (mass) of the classical physics and the scale of energy (frequency) of the quantum mechanics are the same is the Planck time. Thus eq. (10) holds true at the Planck time.

Since the change of length is defined only in the direction  $x$ , the evolution of the density of the matter is given by

$$\rho \propto E a_x^{-1} \propto a_x^{-2} \propto t^{-2}. \quad (14)$$

Thus the evolution of the density of the matter is defined by the same law both within the framework of classical physics (9) and within the framework of quantum mechanics (14).

At  $t = \text{const}$ , the density of the relativistic matter is expressed via the scale of length and correspondingly via the temperature as

$$\rho \equiv a^{-4} \equiv T^4. \quad (15)$$

Due to the evolution of the density of the matter (14), the temperature defined from the density of the relativistic matter changes with time as

$$T \equiv \rho^{1/4} \propto a^{-1/2} \propto t^{-1/2}. \quad (16)$$

## 2.4 The radial, angular diameter and luminosity distances

The law for the velocity along the scale of length (2) leads to the invariance of the relations between the distances under time transformation  $t \rightarrow t + \text{const}$ ,  $r_1/r_2 = \text{const}$ . Since the scales of length and mass are bounded by the linear relation (7), from this it follows the invariance of the relations between the masses under time transformation  $t \rightarrow t + \text{const}$ ,  $m_1/m_2 = \text{const}$ . On the other hand, the scale of energy (frequency) evolves in accordance with eq. (12), and the density of the matter evolves in accordance with eqs. (9), (14). From this one observes the universe with the stationary euclidean space and the stationary distributed matter. The evolution of the universe manifests itself in the change of the scale of energy (frequency) or in the change of the density of the matter with time.

Let an observer view the universe at the modern time  $t_0$  with the modern size  $a_0$ . Due to the evolution of the scale of energy (12), the energy of photon decreases with time  $E \propto 1/t$  and correspondingly with the distance covered by photon  $E \propto 1/r$ . In the model under consideration, the radial and angular diameter distances are the same. In view of eq. (12), the distance, radial or angular diameter, is given by

$$\frac{r}{a_0} = \frac{\Delta E}{E} = \frac{z}{1+z}. \quad (17)$$

Eq. (17) describes the Hubble law, with the redshift being caused by the evolution of the scale of energy. In view of eq. (14), the intrinsic luminosity of the object  $L$  and the observed flux  $F$  are related as

$$F = \frac{L}{4\pi r^2(1+z)^2} = \frac{L}{4\pi r_L^2} \quad (18)$$

where  $r_L$  is the luminosity distance. The luminosity distance is expressed via the angular diameter distance as

$$r_L = r(1+z) = a_0 z. \quad (19)$$

The change of the energy of photon with redshift can be registered in two ways, as the change of the frequency of photon  $\omega \propto 1+z$ , with the photon emission rate being fixed

$1/t = \text{const}$ , or as the change of the photon emission rate  $1/t \propto 1 + z$ , with the frequency of photon being fixed  $\omega = \text{const}$ . While measuring the photon flux through the photometric band, one deals with the later case. From this K-corrections which arise due to the change of the frequency of photon with redshift have no meaning.

## 2.5 Rescaling due to relativistic and quantum additions

Eq. (17) holds for the local region  $r \ll a_0$ . Taking into account the relativistic addition, rewrite eq. (17) in the form

$$\frac{r}{a_0} = \frac{z}{1+z} - \frac{1}{2} \left( \frac{z}{1+z} \right)^2. \quad (20)$$

From eq. (20) it follows that, at  $z \rightarrow 0$ ,  $r \propto a_0$ , whereas, at  $z \rightarrow \infty$ ,  $r \rightarrow 1/2a_0$ . Thus due to relativistic effects the distance in the local region  $z \rightarrow 0$  differs by a factor of 2 from the distance in the global region  $z \rightarrow \infty$ . Introduce the local scale of time  $\tau$  which is related to the global scale of time as

$$\tau = \frac{1}{2}t. \quad (21)$$

In view of eq. (8), the local scale of time defines the local scale of length and the local scale of mass

$$a(\tau) = \frac{1}{2}a(t). \quad (22)$$

$$m(\tau) = \frac{1}{2}m(t). \quad (23)$$

In the quantum electrodynamics, the scale of mass is normalized with the electromagnetic coupling  $\alpha$

$$m \propto \alpha. \quad (24)$$

Due to quantum additions, the electromagnetic coupling  $\alpha$  changes with momentum transferred. Since the energy scale in the universe evolves it is necessary to take into account the change of the scale of mass and equivalently of the scales of length and time due to the change of the electromagnetic coupling  $\alpha$ .

## 3 Predictions and observational constraints

### 3.1 Constraints from the modern age and Hubble parameter

Determine the modern age of the universe from eq. (16) taking into account the change of the electromagnetic coupling  $\alpha$  due to quantum additions

$$t_0 = \alpha_0 t_{Pl} \left( \frac{T_{Pl}}{T_0} \right)^2 \quad (25)$$

where  $T$  is the temperature of the cosmic microwave background radiation, the subscript  $Pl$  corresponds to the Planck period, the subscript 0 corresponds to the modern period. Here it is taken into account that  $\alpha_{Pl} = 1$ . Calculations yield the modern age of the universe

$t_0 = 33.7$  Gyr. This value corresponds to the global scale. In view of eq. (21), the local value for the modern age of the universe is  $\tau_0 = t_0/2 = 16.9$  Gyr, whereas the experimental value is  $\tau_0 = 14 \pm 2$  Gyr [2]. The modern age of the universe  $\tau_0 = 16.9$  Gyr yields the modern Hubble parameter  $H_0 = 1/\tau_0 = 58$  km/s/Mpc, whereas the experimental value is  $H_0 = 60 \pm 10$  km/s/Mpc [3].

### 3.2 Constraints from the magnitude-redshift relation

Two independent groups [4] have been published data on the redshift-luminosity relation of SN Ia at redshifts  $z \sim 0.4 - 0.8$ . The difference in apparent magnitudes of objects with the same intrinsic luminosity but at different redshifts provides a valuable, classical cosmological test on the evolution law of the universe. The expected difference in apparent magnitudes is given by  $\Delta m \equiv 5 \log[r_L(z_2)/r_L(z_1)]$ . In view of eq. (19), for  $z_1 = 0.4$  and  $z_2 = 0.8$ ,  $\Delta m = 1.5$ . As shown above, applying K-corrections have no meaning, so it is necessary to use the magnitudes without K-corrections. According to Perlmutter et al. [4], without applying K-corrections  $\Delta m \approx 1.5 \pm 0.2$  that favours the model of the universe under consideration. That is constraints from the magnitude-redshift relation of SN Ia favour the linear evolution law  $a = t$ .

### 3.3 Constraints from primordial nucleosynthesis

Authors of [5] investigated constraints on power-law models of the universe, in particular, from primordial nucleosynthesis. The time-temperature relation  $T^{-1} \leq t^{0.58}$  is obtained from the condition that at the beginning of nucleosynthesis when  $T \sim 80$  keV the age of the universe should be less than the lifetime of neutron  $t \leq 887$  s. The inferred primordial abundances of helium-4 [6] and deuterium [7] require  $T^{-1} \sim t^{0.55}$  [5]. Note that here  $T$  is the temperature of the cosmic microwave background radiation,  $t$  is the age of the universe which are related to their modern values.

Since the temperature of the cosmic microwave background radiation is defined by the energy density, according to eq. (16) the time-temperature relation is  $T^{-1} \sim t^{0.5}$ . This relation is obtained with the use of the global scales. Constraints from primordial nucleosynthesis limited by the lifetime of neutron  $T^{-1} \leq t^{0.58}$  and those limited by the inferred primordial abundances of helium-4 and deuterium  $T^{-1} \sim t^{0.55}$  are obtained with the use of the local scales. It is necessary to take into account the transition factors from the local scales to the global scales for the modern age of the universe and for the weak interaction rates in the primordial plasma which define the lifetime of neutron and the time to establish the neutron-proton equilibrium. In view of eq. (22), the local-global transition factor for the modern age of the universe is equal to 2. The weak interaction rates depend on energy as  $\sim E^5$ . In view of eq. (23), the local-global transition factor for the energy is equal to 2, and correspondingly the local-global transition factor for the weak interaction rates is equal to  $2^5 = 32$ . Taking into account the local-global transition factors for the modern age of the universe and for the weak interaction rates, the power index in the relation  $T^{-1} \sim t^{0.5}$  increases by the value  $\sim 0.05$ . Thus this time-temperature relation is in agreement with the constraints from primordial nucleosynthesis limited by the lifetime of neutron  $T^{-1} \leq t^{0.58}$

and with those limited by the inferred primordial abundances of helium-4 and deuterium  $T^{-1} \sim t^{0.55}$ .

### 3.4 The fractal galaxy distribution

The galaxy distribution can be viewed as a fractal (see e.g. [8]), with the average number of galaxies within radius  $R$  from any given galaxy being given by

$$N(< R) \propto R^D \quad (26)$$

where  $D$  is the fractal power index. The conventional point of view is that, on scales  $< 20 h^{-1}$  Mpc, galaxies obey  $D \approx 1.2 - 2.2$ . On scales  $> 20 h^{-1}$  Mpc, the fractal power index increases with scale towards the value  $D = 3$  on scales of about  $100 h^{-1}$  Mpc. On the contrary authors of [9],[10] claimed that galaxies have a fractal distribution with constant  $D \approx 2$  on all scales. Having based on the analysis of the galaxy number counts  $N(< m)$ , they noted that galaxy evolution, modification of the Euclidean geometry and the K-corrections are not very relevant in the range of the present data. Authors of [11] following [10] reanalyzed the ESP survey and suggested the value  $D \simeq 3$ . But they reproduced the result of [10] for the case of neglecting K-corrections and using euclidean metric with the luminosity distance  $r_L = a_0 z$ .

In the universe under consideration, the fractal galaxy distribution arises due to the evolution of the scale of mass with time. Write the average number of particles in the form

$$N(< R) \propto \rho R^3 / m \quad (27)$$

where  $m$  is the mass of the particle. Define the fixed matter density  $\rho = \text{const}$  at  $t = \text{const}$  in all the regions of the universe. In view of eq. (8), the scale of mass changes with time as  $m \propto t$ . Hence the scale of mass changes with radius as  $m \propto R$ . Taking into account the evolution of the scale of mass, the average number of particles takes the form

$$N(< R) \propto R^2. \quad (28)$$

Thus the universe under consideration predicts the fractal galaxy distribution with the power index  $D = 2$ . Note that the case of neglecting K-corrections and using euclidean metric with the luminosity distance  $r_L = a_0 z$  for which the ESP survey data show  $D \approx 2$  [11] corresponds to the universe under consideration.

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